# Verifying trigonometric identities

Process: make one side look exactly like the other using a combination of trigonometric identities and algebra. You can work with only one side at a time.

Some popular strategies:

1. Algebra techniques utilized — Example type
   a. "Foil"ing-example 1 — $(\cot(x) - \csc(x))(\cos(x) + 1) = -\sin(x)$
   b. "Foil"ing- example 2 — $\frac{[\sin(t)+\cos(t)]^2}{\sin(t)cos(t)} = 2 + \sec(t)\csc(t)$
   c. Distribution — $\sec(t)\csc(t)\,[\tan(t) + \cot(t)] = sec^2(t) + csc^2(t)$
   d. Common Denominator — $2\sec(x) = \frac{1}{\sec(x)+\tan(x)} + \frac{1}{\sec(x)-\tan(x)}$
2. Use the Conjugate — $\frac{1-\cos(x)}{\sin(x)} = \frac{\sin(x)}{1+\cos(x)}$
3. Substitution of Trig Identity — $\sin^2(\mathrm{x}) + \cos^2(\mathrm{x}) + \tan^2(\mathrm{x}) = \sec^2(\mathrm{x})$
4. Turn all functions into sin(x) or cos(x) — $\frac{\cos(x)}{\sec(x)} + \frac{\sin(x)}{\csc(x)} = 1$

If all else fails, turn everything into sine x and cosine x and see what happens! Usually there is lots of algebra between using the trig functions. You have to be very familiar with the basic identities.

**Basic Identities:**

$$\sec(x) = \frac{1}{\cos(x)};\ \csc(x) = \frac{1}{\sin(x)};\ \cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)};\ \tan(x) = \frac{1}{\cot(x)} = \frac{\sin(x)}{\cos(x)}$$

$$\sin^2(\mathrm{x}) + \cos^2(\mathrm{x}) = 1;\ 1 + \cot^2(\mathrm{x}) = \csc^2(\mathrm{x});\ \tan^2(\mathrm{x}) + 1 = \sec^2(\mathrm{x})$$ (*triplets)

When working with the triplets, the last two can be obtained by dividing the first either by $sin^2(x)$ or $cos^2(x)$.

*Any of the triplets could be changed to isolate a different term. Such as:*

$$cos^2(x) = 1 - sin^2(x) \text{ or } 1 = sec^2(x) - tan^2(x)$$

# Examples

Detailed solutions, (remember, work with only one side until it looks like the other)

1. $[\cot(x) - \csc(x)][\cos(x) + 1] = -\sin(x)$ **work with the more complicated side**

$= \cot(x)\cos(x) + \cot(x) - \csc(x)\cos(x) - \csc(x)$ foil out groups

$= \frac{\cos(x)}{\sin(x)}[\cos(x)] + \frac{\cos(x)}{\sin(x)} - \frac{1}{\sin(x)}[\cos(x)] - \frac{1}{\sin(x)}$ use identities to change to sin(x) & cos(x)

$= \frac{cos^2(x)}{\sin(x)} + \frac{\cos(x)}{\sin(x)} - \frac{cos(x)}{\sin(x)} - \frac{1}{\sin(x)}$ then combine like terms

$= \frac{cos^2(x)-1}{\sin(x)}$ use identity to simplify

$= \frac{(1 - sin^2(x)) - 1}{\sin(x)}$

$= \frac{-sin^2(x)}{\sin x}$ then, reduce

$= -\sin(x)$ ✓

2. $\frac{(\sin(t)+\cos(t))^2}{\sin(t)\cos(t)} = 2 + \sec(t)\csc(t)$ working with left side

$= \frac{sin^2(t)+2\sin(t)\cos(t)+cos^2(t)}{\sin(t)\cos(t)}$ Foil out the top

$= \frac{[\sin^2(t)+\cos^2(t)]+2\sin(t)\cos(t)}{\sin(t)\cos(t)}$ groups so you can use an identity to simplify

$= \frac{1+2\sin(t)cost(t)}{\sin(t)\cos(t)}$ next, separate into two terms since answer has two

$= \frac{1}{\sin(t)\cos(t)} + \frac{2\sin(t)\cos(t)}{\sin(t)\cos(t)}$ ✓ reduce as possible

$= \frac{1}{\sin(t)\cos(t)} + 2$ use identities to simplify the first term

$= \csc(t)\sec(t) + 2$ ✓

3. $\sec(t)\csc(t)[\tan(t) + \cot(t)] = sec^2(t) + csc^2(t)$ **(working with left side again)**

$= sec(t)csc(t)tan(t) + sec(t)csc(t)cot(t)$ distribute

$= \frac{1}{\cos(t)}\frac{1}{\sin(t)}\frac{\sin(t)}{\cos(t)} + \frac{1}{\cos(t)}\frac{1}{\sin(t)}\frac{\cos(t)}{\sin(t)}$ re-write with reciprocals

$= \frac{1}{cos^2(t)} + \frac{1}{sin^2(t)}$ cancel like terms

$= sec^2(t) + csc^2(t)$ ✓ use reciprocals to reduce

**4.** $\mathbf{2\sec(x) = \frac{1}{\sec(x)+\tan(x)} + \frac{1}{\sec(x)-\tan(x)}}$ **working with right side**

$= \frac{1[sec(x)-tan(x)]}{[sec(x)+tan(x)][sec(x)-tan(x)]} + \frac{1[sec(x)+tan(x)]}{[sec(x)+tan(x)][sec(x)-tan(x)]}$ get a common denominator

$= \frac{sec(x)-tan(x)+sec(x)+tan(x)}{[sec(x)+tan(x)][sec(x)-tan(x)]}$ combine into 1 fraction

$= \frac{2sec(x)}{sec^2(x)-tan^2x}$

combine like terms/foil

= $2sec(x)$ ✓

**5.** $\mathbf{\frac{1-\cos(x)}{\sin(x)} = \frac{\sin(x)}{1+\cos(x)}}$ **choose either side**

$= \frac{[1-\cos(x)][1+\cos(x)]}{\sin(x)[1+\cos(x)]}$ multiple by conjugate

$= \frac{1-cos^2(x)}{\sin(x)[1+\cos(x)]}$ FOIL the top

$= \frac{sin^2(x)}{\sin(x)[1+\cos(x)]}$ substitute the identity

$= \frac{\sin(x)}{1+\cos(x)}$ ✓ reduce fraction

**6.** $\mathbf{\sin^2(x) + \cos^2(x) + \tan^2(x) = \sec^2(x)}$ **choose the left side, more pieces**

$= 1 + tan^2(x)$ use identity

= $sec^2(x)$ ✓ use identity

7. $\frac{\cos(x)}{\sec(x)} + \frac{\sin(x)}{\csc(x)} = 1$ choose the left side to work with

$= \frac{\cos(x)}{\frac{1}{\cos(x)}} + \frac{\sin(x)}{\frac{1}{\sin(x)}}$ using identities/reciprocals

$= cos(x) \cdot \left(\frac{\cos(x)}{1}\right) + sin(x)\left(\frac{sin(x)}{1}\right)$ invert to convert division to multiplication

$= cos^2(x) + sin^2(x)$ use identity

$= \mathbf{1}$ ✓

***Watch for identities and algebra steps to be intermingled!***

**Check out the You try's on the back side for some more practice!**

**You Tries:**

1. $(\csc(x)+1)(\csc(x)-1)= \ cot^2(x)$; recommended strategy - FOIL

2. $\frac{1-\csc(x)}{1+\csc(x)}=\frac{\sin(x)-1}{\sin(x)+1}$; recommended strategy: conjugate

3. $\frac{\sin(t)}{1-\cos(t)}-\frac{1+\cos(t)}{\sin(t)}=0$; recommended strategy: common denominator

4. $\frac{\cos(x)}{\sin(x)+1}-\frac{\cos(x)}{\sin(x)-1}=2\csc(x)$; recommended strategy: common denominator

5. $\frac{\tan(x)}{\sin x-2\tan(x)}=\frac{1}{\cos(x)-2}$; recommended strategy: change functions to sin(x) and cos(x)