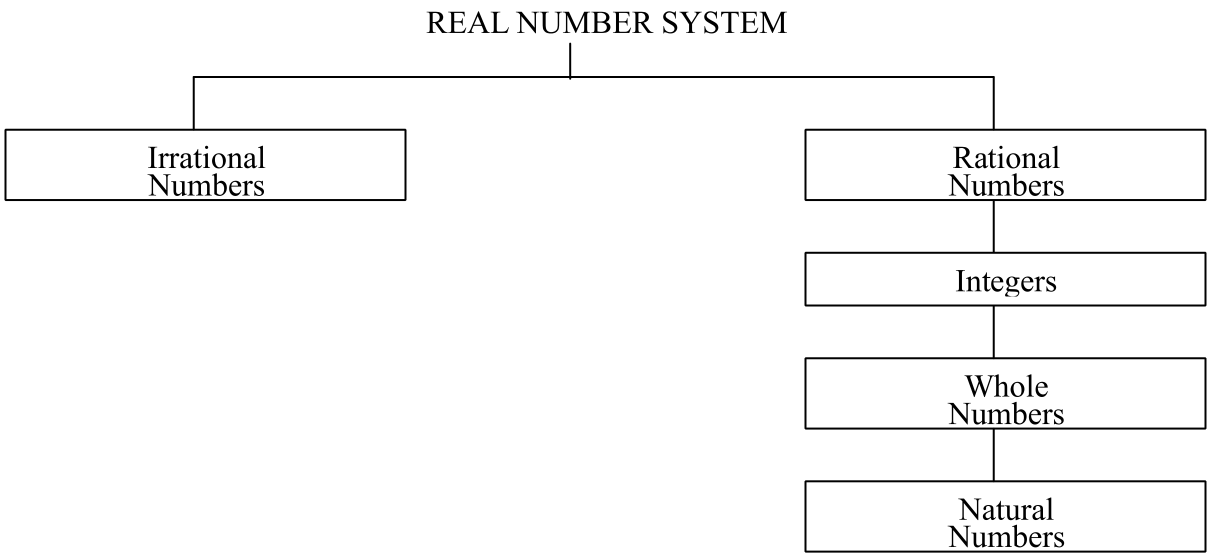
Properties of Real Numbers

We will introduce some properties of real numbers which will be very important throughout your algebra courses. It is important that you understand each property and how it works. These properties apply to all numbers in the real number system.



Rational numbers are real numbers that can be written as the ratio of two integers. Irrational numbers cannot. As all irrational numbers are non-repeating and non-terminating decimals, and all rational numbers are repeating or terminating decimals, an easy way to think of the real number system is as the set of all decimal numbers. These properties then apply to any number which can be written as a decimal.

We will separate the properties into properties of addition and properties of multiplication.

> The Commutative Property of Addition
>
> If a and b are real numbers, then $a + b = b + a$
>
> This property of addition allows us to change the <u>order</u> in which we add two numbers without changing the answer

For example, we know that $3 + 5 = 8$ and $5 + 3 = 8$ also.

$a + b = b + a$	$a + b = b + a$
$3 + 5 = 5 + 3$	$-9 + 4 = 4 + (-9)$
$8 = 8$	$-5 = -5$

NOTE that a and b can stand for <u>any</u> two numbers.

The Associative Property of Addition

If a, b and c are real numbers, then $(a + b) + c = a + (b + c)$.

This property of addition allows us to change the way we group the numbers that are being added, without changing the answer.

$$(a + b) + c = a + (b + c)$$
$$(3 + 2) + 4 = 3 + (2 + 4)$$
$$5 + 4 = 3 + 6$$
$$9 = 9$$

$$(a + b) + c = a + (b + c)$$
$$[3 + (-7)] + -(2) = 3 + [-7 + (-2)]$$
$$-4 + (-2) = 3 + (-9)$$
$$-6 = -6$$

NOTE that a, b and c can be any three numbers. Be careful that you can distinguish between these two properties of addition. It is not always easy.

EXAMPLE: $(3 + 2) + 7 = 7 + (3 + 2)$

Is this an example of the Associative Property of Addition or the Commutative Property of Addition? Look carefully! What has changed? At first glance you may think it is the Associative Property, but don't let those parentheses fool you! If you look carefully, you will see that the group HAS NOT CHANGED. The 3 and the 2 are still grouped together. What HAS changed is the order. It was 3, 2, 7, and it became 7, 3, 2. This is an example of the Commutative Property of Addition!

$$a + b = b + a$$

$$(3 + 2) + 7 = 7 + (3 + 2)$$

The Addition Property of Zero

If a is a real number, then $a + 0 = 0 + a = a$

This property means that we can add zero to any number without changing the value of that number.

Zero is also called the Identity for addition. This is because zero is the only number which can be added without changing the value of the other addend. Adding zero to a number does not change that number. NOTE that a can be any real number.

The Inverse Property of Addition

If a is a real number, then, $a + (-a) = -a + a = 0$

This property means that whenever we add a number and its opposite, the sum will always be zero. The opposite of a number is also called the additive inverse of that number.

$a + -(a) = -a + a = 0$ $a + -(a) = -a + a = 0$

$5 + (-5) = -5 + 5 = 0$ $-3 + [-(-3)] = [-(-3)] + (-3) = 0$

NOTE that *a* can be any real number.

If you have a good understanding of the properties of addition, the Commutative and Associative Laws of Multiplication will be easy for you. They are the same as the laws for addition except that they apply to the operation of multiplication.

The Commutative Property of Multiplication

If *a* and *b* are real numbers, then $a \cdot b = b \cdot a$

This property allows us to change the order of two numbers when we are multiplying without changing the answer.

$a \cdot b = b \cdot a$ $a \cdot b = b \cdot a$

$3 \cdot 5 = 5 \cdot 3$ $-6 \cdot 2 = 2 \cdot (-6)$

$15 = 15$ $-12 = -12$

NOTE that *a* and *b* can be any two numbers.

The Associative Property of Multiplication

If *a*, *b*, and *c* are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

This property allows us to change the way factors are grouped in a multiplication problem.

$(3 \cdot 2) \cdot 8 = 3 \cdot (2 \cdot 8)$ $(-3 \cdot 4) \cdot 2 = -3 \cdot (4 \cdot 2)$

$6 \cdot 8 = 3 \cdot 16$ $-12 \cdot 2 = -3 \cdot 8$

$48 = 48$ $-24 = -24$

NOTE that *a*, *b* and *c* can be any three numbers.

As with addition, be sure that you can distinguish between the two laws of multiplication.

$$(3 \cdot 2) \cdot 5 = 5 \cdot (3 \cdot 2)$$

What has changed? Order or grouping? The grouping is still the same. It is the order that has changed, so this is an example of the Commutative Property of Multiplication.

The Multiplication Property of Zero

If *a* is a real number, then $a \cdot 0 = 0 \cdot a$

This means that when any number is multiplied by zero, the product is always zero.

$6 \cdot 0 = 0 \cdot 6 = 0$: note that a can be any number.

The Multiplication Property of One

If a is a real number, then $a \cdot 1 = 1 \cdot a = a$

This property means that we can multiply any number by one without changing the value of that number.

$$2 \cdot 1 = 1 \cdot 2 = 2$$

One is also called the *identity* for multiplication. This is because "one" is the only number that you can multiply by so that the value of the other number does not change. Multiplying a number by one does not change that number. NOTE that *a* can be *any* number.

The Inverse Property of Multiplication

If a is a real number and a ≠ 0, $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$

This means that the product of any number and its reciprocal (also called the multiplicative inverse) will always be the number one.

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$
$$2 \cdot \frac{1}{2} = \frac{1}{2} \cdot 2 = 1$$

NOTE that *a* can be any number except zero. We cannot have zero in the denominator of a fraction because division by zero is undefined.

The last property we will look at involves both multiplication and addition.

The Distributive Property

If a, b and c are real numbers, then $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$

This property allows us to either add inside the parentheses first and then multiply that sum, or to multiply each number inside the parentheses by the number on the outside, and then add the products. The answer will be the same.

$$a(b + c) = ab + ac$$
$$3(2 + 4) = 3(2) + 3(4)$$
$$3(6) = 6 + 12$$
$$18 = 18$$

NOTE that a, b and c can be any three numbers. This property will be particularly helpful when we need to remove parentheses from expressions containing variables.

III. EXERCISES: Identify the property which justifies each statement.

a. $-5 + 5 = 0$

b. $3(2 + 6) = 3(2) + 3(6)$

c. $-4 + 10 = 10 + (-4)$

d. $(-2 + 7) + 4 = -2 + (7 + 4)$

e. $0 \cdot 6 = 0$

f. $\frac{1}{3} \cdot 3 = 1$

g. $(-2)(-4) = (-4)(-2)$

h. $-8 + 0 = -8$

i. $(3 \cdot 7) \cdot 9 = 3 \cdot (7 \cdot 9)$

j. $12 \cdot 1 = 12$

KEY:

b. Distributive Property
c. Commutative Property of Addition
d. Associative Property of Addition
e. Multiplication Property of Zero
f. Inverse Property of Multiplication
g. Commutative Property of Multiplication
h. Addition Property of Zero
i. Associative Property of Multiplication
j. Multiplication Property of One