TSC | WILLIAM D. LAW, JR. LEARNING COMMONS

Factoring Summary

Basic Process/Options/Types

1. Always check to see if there is a Greatest Common Factor (GCF): $\boldsymbol{a}x^2$+ **a**bx + **a**c = **a** (x^2+ bx + c).
2. Factor by grouping if you have 4 terms. (see example below)
3. Form: x^2+ bx + c, first or last term has a coefficient of 1. Find factor pairs of the other coefficient.
4. Form: ax^2+ bx + c, no special pattern, use trial and error or AC method. See below.
5. Form: $x^2 - y^2$= (x – y) (x + y), Difference of two squares.
6. Form: x^2+ y^2= cannot be factored with integer factors, Sum of two squares.
7. Form: $x^3 - y^3$= (x – y) (x^2+ x y + y^2); Difference of cubes. The first sign matches the original.
8. Form: x^3+ y^3= (x + y) ($x^2 -$ x y + y^2); Difference of cubes. The first sign matches the original.

Examples: Directions- factor completely.

1. $3x^2$+ 9x + 15; Has a **GCF** or 3, so factor that out first. $\mathbf{3}$($\boldsymbol{x^2}$**+ 3x + 5).** This cannot be factored again.

2. $3x^3 + 2x^2 -$ 6x – 4; this has **4 terms** so use grouping: $(3x^3 + 2x^2) + (-6x - 4)$,
then look for common terms in each group: $x^2(3x + 2) - 2$ (3x + 2).
Next, look for the group that is a new GCF and factor it out. $\boldsymbol{(3x + 2)(x^2 - 2)}$**.**
If either factor can be factored further, keep going.

3. x^2+ 4x – 12; Since the **first coefficient is a 1**, we can use trial and error by looking at the factor pairs of c.
a = 1: factors→(1,1), c = -12: factors → (1, -12), (-1, 12), (2, -6), **(-2,6),** (3, -4), (-3,4). Looking for a combination that adds up to our b: 4. Thus our factors are **(x – 2) (x + 6).**

4. $3x^2$+ 2x – 8. Once we determine there is not a GCF, we can either use **Trial and Error or the AC method**.
Trial and Error: look at factor pairs of each of the A and C terms. A = 3 => {1,3}. C = -8 => {1,-8}, {-1,8}, {2, -4}, {-2,4}. By using various combinations, it is found that the answer is **(3x – 4)(x + 2).**

   AC method: Since all quadratics are of the form: ax^2+ bx + c, we can use the AC method. Identify a, b, c. This method requires that the factor choices start with A $\cdot$ C. This product should add up to our B. Then we use the grouping method to complete.

   $3x^2$+ 2x – 8. a = 3, b = 2 and c = -8. Thus A $\cdot$ $C = -24$, b = 2.

1	-24	Sum = -23
-1	24	Sum = 23
2	-12	Sum = -10
-2	12	Sum = 10
3	-8	Sum = -5
-3	8	Sum = 5
4	-6	Sum = -2
-4	**6**	**Sum = 2**

   So, the choice we want is -4, 6. The middle term is broken into 2 pieces using these coefficients.
   $3x^2$+ **2x** – 8 transforms into: $3x^2$**- 4x + 6x** – 8. Now, we can use grouping since we have 4 terms.
   $3x^2$- 4x + 6x – 8 => $(3x^2 -$ 4x) + (6x – 8). Factor out the GCF in each group next. x (3x – 4) + 2(3x – 4). We now have a new GCF (the group left behind after the original GCF is factored out). $\boldsymbol{(3x - 4)(x + 2)}$**.**

5. $9x^2 - 36y^2$. This example has a GCF, so start with that. $9(x^2 - 4y^2)$. Now since we only have 2 terms that both can be expressed as a perfect square, we can factor that in a special way. The factors look exactly alike except for their sign. Don't forget the GCF we factored out previously. **9 (x – 2y) (x + 2y).**

6. $9x^2 + 36y^2$. This example has a GCF, so start with that. $9(x^2 + 4y^2)$. We cannot continue with integer factors past this.

7. $8x^3$- $27y^3$. It is a good step to re-write each term as a group to the third before applying the formula. $(2x)^3 - (3y)^3 =$ (2x – 3y) ($4x^2$+ 6x y + $9y^2$); Cubes are allowed one negative sign. The first group follows the sign of the original problem. If it is not used then, it is inserted before the middle term in the second group. The second group cannot be factored further using integer factors.

8. $8x^3$+ $27y^3$. $(2x)^3 + (3y)^3 =$ (2x + 3y) ($4x^2 -$ 6x y + $9y^2$). Notice the original group's sign follow the original problem, thus the one negative sign required has been inserted into the second factor into the correct position.

Sign Hints:

If the trinomial has the form: $\boldsymbol{ax^2}$**+ bx + c**, then the factored form will also have all positives, $(px + m)(qx + n)$

If the trinomial has the form: $\boldsymbol{ax^2}$**- bx + c**, then the factored form will also have two negatives, $(px - m)(qx - n)$

If the trinomial has the form: $ax^2 \pm$ bx - c, then the factored form will have 1 negative and 1 positive, $(px - m)(qx + n)$ or $(px + m)(qx - n)$.