Steps to Solve a Related Rate of Change Problem

1. List the given information and identify the missing one. Draw a shape if it is possible.
2. Use the given function or **<u>any given information</u>** to find a relationship between the involved variables.
3. Calculate the relevant derivative and substitute in the given information in the final equation.

Single rate of change: those problems work with one variable and keep all others constant.

Sample problem- Two Dimensional

When heating a square piece of metal, it is observed that the sides increase as the heating process goes on.

a) Determine the shape and any relevant formulas from geometry.
b) Find the relationship between the rate of change in the Area of the square and the rate of change of its sides.
c) Suppose at a certain time, the side was 4 meters, and the rate of change was 1.5 m/s, find the rate of change of the Area of the square.

Set-up solution.

1. Find the relationship between the change in the Area and the change in the side:
2. Information: The side of the square = s, the change in the side is ds/dt, the Area of the square A= s^2, and the change in the Area is dA/dt.
3. Relationship between variables:

$$A = s^2, then,$$
$$dA/dt = 2sds/dt$$

a) The rate of change of the Area, when S= 4, ds/dt = 1.5, dA/dt?

$$dA/dt = 2sds/dt.$$
$$dA/dt = 2(4)(1.5)$$
$$dA/dt = \mathbf{12\ m^\wedge 2/s}$$

Sample problem- Three Dimensional

The radius of a cylinder is increasing at rate of 5 meters per second. If the height is constant and measures 10 meters, find the rate of change of the Volume when the radius reaches 12 meters.

Set up to solution $\boldsymbol{h}$

a. Information: dr/dt= 5m/s; h= 10 m; r=12 m; dv/dt?
b. Relationship between Volume and radius is given through the formula $V = \pi \cdot r^2 \cdot h$

Find the derivative of V(t)

$$\frac{dv}{dt} = 2r\frac{dr}{dt} * h * \pi$$

$$\frac{\boldsymbol{dv}}{\boldsymbol{dt}} = 2(12\ m)(5\frac{\mathrm{m}}{\mathrm{s}})(10\ m) * 3.14$$

$$\mathbf{3769.91 m^\wedge 3/s}$$

Multiple Rates of Change: Those types of problems work with more than one variable.

Sample problems

The radius of a cylinder is increasing at a rate of 1.5 meters per hour, and the height of the cylinder is decreasing at a rate of 5 meters per hour. At a certain instant, the base radius is 6 meters, and the height is 12 meters. What is the rate of change of the Volume of the cylinder at the instant?

Set up solution:

c. hInformation: dr/dt= 1.5m/s; h= 12 m; dh/dt= -5m/s; r=6 m; looking for $\frac{\mathbf{dv}}{\mathbf{dt}}$.

$\boldsymbol{V = \pi r^2 h}$, next take the derivative both with respect to r and h.

$$\frac{\mathbf{dv}}{\mathbf{dt}} = 2\frac{r\,dr}{dt} * h * \pi + \pi r^2 \frac{dh}{dt}$$

$$\frac{\boldsymbol{dv}}{\boldsymbol{dt}} = 2 * (6\,m) * \left(1.5\frac{\mathrm{m}}{\mathrm{s}}\right) * (12\,m) * 3.14 + (6\,m)^2 * \left(-\frac{5\mathrm{m}}{\mathrm{s}}\right) * 3.14 = \mathbf{113.04}\frac{\boldsymbol{m^3}}{\boldsymbol{s}}$$

You try:

A. The Area of a circle is increasing at a rate of 2-meter square per second. Find the rate of change of the radius when it is 4 meters long.

B. The diameter of a sphere is decreasing at a rate of 5 cm per hour. Find the rate of change of its Volume when the diameter reaches 20 cm. (Volume of a sphere is $V = \frac{4}{3}\pi r^3$)

C. A tank is shaped like an upside-down square pyramid, with a base of 4 m by 4 m and a height of 12 m (see the figure below). How fast does the height increase when the water is 2 m deep if water is being pumped in at a rate of 2/3 m^3/sec? ($V = \frac{1}{3}B * h$) .

Answers

A: .08 m/s
B: 3141.59 cm^3/h
C: 3/2 m/s