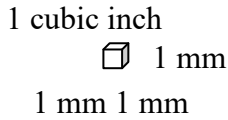
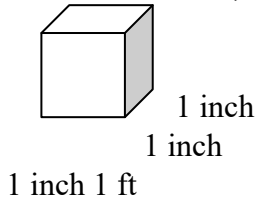


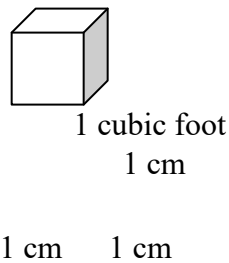
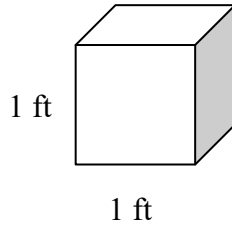
# Volume of Basic Geometric Figures

Volume measures the amount of space inside a closed surface. To measure volume we must use a unit of measurement that fills up space. (The linear unit and the square unit will not meet this requirement.)

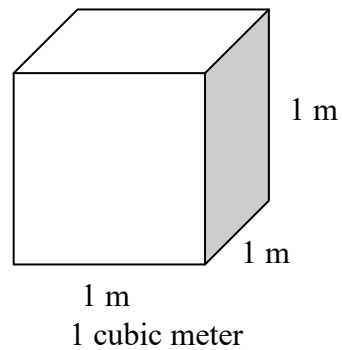
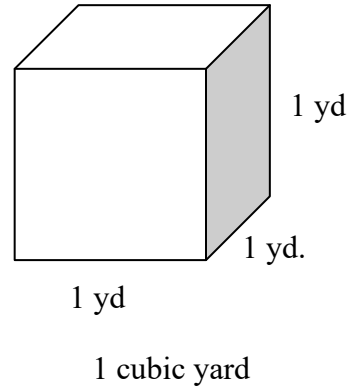
To measure volume, use cubic units such as:



1 cubic millimeter



1 cubic centimeter



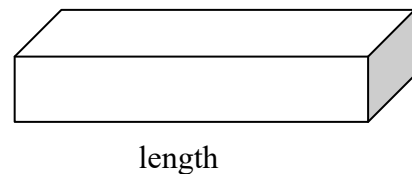
The abbreviations used for these cubic units are:

cubic inch: $\text{in}^3$	cubic foot: $\text{ft}^3$	cubic yard: $\text{yd}^3$
cubic millimeter: $\text{mm}^3$	cubic centimeter: $\text{cm}^3$	cubic meter: $\text{m}^3$

FORMULAS: You should memorize the volume formulas for these common geometric solids.

## A. RECTANGULAR SOLID

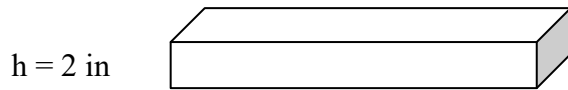
$$\text{VOLUME} = \text{LENGTH} \times \text{WIDTH} \times \text{HEIGHT}$$



### EXPLANATION:

Length  $\times$  width gives the area of the base, but it also tells the number of cubes that are on the

first layer. If we multiply by the number of layers (the height tells this), we'll have the number of cubic units in the rectangular solid.



w = 4 in

L = 9 in **EXAMPLE:** Find the volume of a rectangular solid

which has a length of 9 inches, a width of 4 inches and a height of 2 inches.

$$\begin{aligned} \text{VOLUME} &= \text{LENGTH} \times \text{WIDTH} \times \text{HEIGHT or } V = l w h \\ &= 9\text{in} \times 4\text{in} \times 2\text{in} \\ &= 72 \text{ in}^3 \end{aligned}$$

## B. CUBE

The faces of a cube are squares.

$$\text{Volume} = \text{Length} \times \text{width} \times \text{height}$$

becomes

$$\text{Volume} = \text{side} \times \text{side} \times \text{side}$$

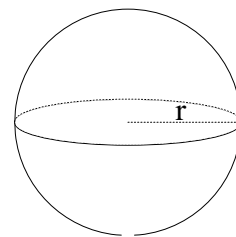
$$\text{or Volume} = (\text{side})^3 \text{ or } V = s^3$$

**EXAMPLE:** Find the volume of a cube which is 8 cm on each side.

$$\begin{aligned} \text{Volume} &= (\text{side})^3 \\ &= (8 \text{ cm})^3 \\ &= 512 \text{ cm}^3 \end{aligned}$$

## C. SPHERE

$$\begin{aligned} \text{Volume} &= \frac{4}{3} \pi \cdot \text{radius} \cdot \text{radius} \cdot \text{radius} \\ \text{or Volume} &= \frac{4}{3} \pi \cdot (\text{radius})^3 \\ &= \frac{4}{3} \pi r^3 \end{aligned}$$



(There is an explanation for this formula, but it is rather complicated for this lab. Maybe you would accept the challenge to find out why this formula works). You do need to memorize

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

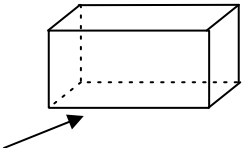
**EXAMPLE:** A sphere has a radius of 6 cm. Find its volume. Use  $\pi = 3.14$

$$\begin{aligned}
&= \frac{4}{3} \cdot \pi \cdot 6^3 \\
&= \frac{4}{3} \cdot (3.14) \cdot (6 \text{ cm})^3 \\
&= \frac{4}{3} \cdot \frac{3.14}{1} \cdot \frac{6^2}{1} \cdot \frac{6}{1} \cdot \frac{6}{1} \\
V &= \frac{4}{3} \cdot 3.14 \cdot 6^2 \cdot 6 \\
&= 904.32 \text{ cm}^3
\end{aligned}$$

D. **CYLINDER:** Volume =  $\pi \times (\text{radius})^2 \times \text{height}$

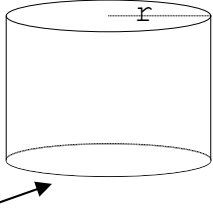
This formula will be easier to remember if you see how it compares with the volume formula of a rectangular solid.

VOLUME  $\underbrace{\hspace{2cm}}$  = Length  $\times$  Width  $\times$  Height  
 Rectangular Solid:  
 (Area of base)  $\times$  height base is a rectangle



CYLINDER  
 Volume  $\underbrace{\hspace{2cm}}$  =  $\pi \times (\text{radius})^2 \times \text{height}$  Cylinder:  
 (Area of base)  $\times$  height or  $V = \pi \cdot r^2 \cdot h$   
 $h^2$

base is a circle



**EXAMPLE:** Find the volume of a cylinder which has a radius of 7 mm and a height of 24 mm.  
 Use  $\pi = \frac{22}{7}$

$$\begin{aligned}
\text{VOLUME} &= \pi \cdot (\text{radius})^2 \cdot \text{height} \\
&= \frac{22}{7} \cdot \frac{7}{1} \cdot \frac{7}{1} \cdot \frac{24}{1} \\
&= 3,696 \text{ mm}^3
\end{aligned}$$

NOTICE: The units were not used in the work. Volume is always measured in cubic units. Look at

the factors.  $\frac{22}{7}$  is just a number, but the other three factors have units of “mm.”

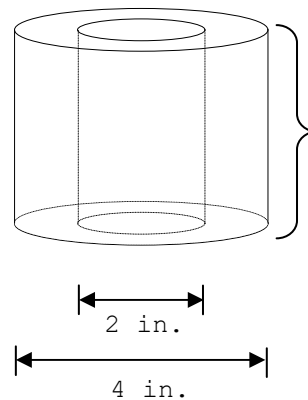
**COMPOSITE GEOMETRIC SOLIDS:**

To find the volume of composite solids, find the volumes of the solids that you recognize. Then add or subtract these volumes to find the volume of the composite solid. It helps to draw the parts and write your plan!

A cylinder is 14 inches high and has a base with a diameter of 4 inches. A cylindrical column with a diameter of 2 inches is cut out of the center of the larger cylinder. Find the volume of the solid that remains. Use  $\pi = \frac{22}{7}$

**PLAN:** Volume of large cylinder – volume of small cylinder

$V = \pi \cdot r^2 \cdot h$ <p>If <math>d = 4</math> in, then</p> $r = \frac{4}{2} = 2 \text{ in}$ $V = \frac{22}{7} \times \frac{2}{1} \times \frac{2}{1} \times \frac{14}{1}$ $= 176 \text{ in}^3$	$V = \pi \cdot r^2 \cdot h$ <p>If <math>d = 2</math> in, then</p> $r = \frac{2}{2} = 1 \text{ in}$ $V = \frac{22}{7} \times 1 \times 1 \times 14$ $= 44 \text{ in}^3$
$\text{VOLUME} = 176 \text{ in}^3 - 44 \text{ in}^3$ $= 132 \text{ in}^3$	



**PROBLEMS:** You should draw the figure and write the formula first.

1. Find the volume of a cylinder that has a height of 12 ft and a radius of 10 ft. Use  $\pi = 3.14$
  
2. Find the volume of a cube that is 6 inches on each side.
  
3. Find the volume of a rectangular solid that is 4.6 cm long, 3 cm wide, and 2.8 cm high.
  
4. Find the volume of a sphere that has a radius of 2 inches. Use  $\pi = 3.14$ . Round answer to the nearest tenth of a cubic inch.

5. Find the volume of a can that is 8 inches high and has a diameter of 5 inches. Use  $\pi = 3.14$ .

6. Find the volume of a refrigerator that is 5 ft high,  $2\frac{1}{2}$  ft deep and 3 feet wide. (The answer will not give the storage space because much of this is thickness of its walls, etc.)

**ANSWER KEY:**

1.  $3768 \text{ ft}^3$

2.  $216 \text{ in}^3$

3.  $38.64 \text{ cm}^3$

4.  $33.5 \text{ in}^3$  (rounded)

5.  $157 \text{ in}^3$

6.  $37\frac{1}{2} \text{ ft}^3$

If you use your calculator, you will use 2.5 ft for  $2\frac{1}{2}$  ft. The answer will be  $37.5 \text{ ft}^3$  which is  $37\frac{1}{2} \text{ ft}^3$ . Fractions are usually used with U.S.units.